Pair correlation function of Wilson loops

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(Received 22 December 1999; published 23 June 2000)

We give a path integral prescription for the pair correlation function of Wilson loops lying in the world volume of D-branes in the bosonic open and closed string theory. The results can be applied both in ordinary flat spacetime in the critical dimension d or in the presence of a generic background for the Liouville field. We compute the potential between heavy nonrelativistic sources in an Abelian gauge theory in relative collinear motion with velocity $v=\tanh(u)$, probing length scales down to $r_{\min}^2=2\pi\alpha' u$. We predict a universal $-(d-2)/r$ static interaction at short distances. We show that the velocity-dependent corrections to the short-distance potential in the bosonic string take the form of an infinite power series in the dimensionless variables $z=r_{\min}^2/r^2$, uz/π, and u^2.

PACS number(s): 11.25.−w, 12.38.Aw, 31.15.Kb

I. INTRODUCTION

It is an old result due to Wilson [1] that the expectation value of the loop observable $W(\mathcal{C})$ in QCD has an effective description at low energies as a sum over random surfaces with a fixed boundary:

$$\langle W(\mathcal{C})\rangle \equiv \left\langle \operatorname{tr} P \exp\left(i \oint_{\mathcal{C}} dX\cdot A\right)\right\rangle \sim Z_{\text{string}}[\mathcal{C}]. \tag{1}$$

$\mathcal{C}$ is the closed world line of a heavy quark-antiquark pair, the angular brackets denote the averaging over gauge field configurations, and the quarks are treated as semiclassical sources in the gauge theory [1]. $Z_{\text{string}}[\mathcal{C}]$ denotes the sum over world sheets terminating on a fixed curve $\mathcal{C}$ in the spacetime of some string theory: an open string amplitude with a macroscopic hole in the world sheet. In the gauge theory $\langle W(\mathcal{C})\rangle \simeq e^{-S_{\text{eff}}[\mathcal{C}]}$ is characterized by an area dependence in the effective action for large loops with widely separated quark and antiquark worldlines, with crossover to a perimeter growth of the effective action for large loops with closely separated world lines at fixed spatial separation [1]. This behavior is characteristic of the long distance effective dynamics of a large class of phenomenological string theories including the Nambu-Goto and Eguchi-Schild strings [1–3]. D-brane backgrounds of critical string theory in flat spacetime and at weak coupling [5] enable a universal and quantitative prediction of the short-distance dynamics of Wilson loops—a regime of QCD that remains largely unexplored by either analytic or lattice techniques. Our result is extracted directly from a covariant path integral computation for the critical Polyakov string with boundaries [4,6], extending techniques developed in earlier works [7,8]. The key ingredient which enables this prediction is its relationship to the vacuum energy computation in string theory: unlike in quantum field theories, the one-loop cosmological constant in critical string theory can be unambiguously normalized, an observation due to Polchinski [7].

Let us review some aspects of the equivalence in Eq. (1) as known from previous work. The leading quantum correction to the area law from the long-distance effective dynamics of the string theory yields an attractive and model-independent $1/r$ term in the static potential between two heavy sources in gauge theory. The static coefficient is the coupling constant and cutoff independent, and was first discovered using the functional methods of Lüscher, Symanzik, and Weisz [9] in a semiclassical quantization of the Eguchi-Schild string. A model-independent argument based on an effective field theory governing the quantum dynamics at large distance scales of a thin flux tube linking the two sources gave the result [10]

$$V(r)=\alpha r+\beta-\frac{\pi}{24}(d-2)\frac{1}{r}+O(1/r^2), \tag{2}$$

where α,β are model-dependent nonuniversal coefficients, and d is the number of spacetime degrees of freedom of the collective coordinate for the thin flux tube. The linear term, signaling confinement, dominates at large separations. The qualitative form of the static potential in Eq. (2) has been extensively confirmed in lattice gauge theory analyses where a high precision measurement of the linear term is easily performed. Recent work in string theory [11] has examined the long-distance effective dynamics of Wilson loops in certain large N gauge theories using a conjectured dual description as an effective limit of the type-IIB string theory in AdS spacetimes [12]. In this limit, gravity decouples from the gauge theory on the D-branes: for large N, there is a clear separation of scales between the gauge theory with effective couplings of $O(g_sN)$ and supergravity with couplings of $O(g_s)$. The leading gravitational corrections to the one-loop amplitude we consider are of $O(g_s^2)$, naturally suppressed at short distances and at weak coupling, even at finite N. On the other hand, the long-distance gauge dynamics of Wilson loops cannot be directly explored by open and closed string theory without taking an appropriate large N limit. At long distances, the bosonic pair correlation function we derive is instead to be interpreted in terms of a different low-energy

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theory—gravity. Duality alters this interpretation considerably in nonperturbative string or M theory. We defer that discussion to a consideration of the superstring [13].

The idea of using the Polyakov string path integral to extend the analytic estimates for the leading quantum corrections to the static potential at long distance into the short-distance regime is due to Alvarez [6]. The implications of a $1/r$ term in the short-distance potential between sources in a non-Abelian gauge theory are discussed in Ref. [15]. At short distances, the notion of a thin flux tube no longer holds but a direct computation of the short-distance potential between sources can be performed in a renormalizable string theory with boundaries. Unlike the Nambu-Goto and Eguchi-Schild strings, the quantization of the bosonic Brink–Di Vecchia–Howe–Deser–Zumino action [14] due to Polyakov [31] treats the world-sheet metric as an independent dynamical field. The action is renormalizable, enabling in principle a closed form analysis of the string functional integral without the need to take an effective long-distance limit. We will carry out that analysis in this paper for *critical* string theory. We show that D-brane backgrounds of open and closed string theory in flat spacetime and at weak coupling [5] provide a calculable framework within which the short-distance behavior of the static potential can be obtained directly from the Weyl invariant string path integral in the critical spacetime dimension. The results can also be adapted to generic conformally invariant backgrounds of string theory with c_m matter fields coupled to a Liouville field with total central charge equal to the critical dimension, following Ref. [16]. The loops are taken to lie in the $(p+1)$-dimensional world volume of a Dp-brane—a hypersurface in a higher-dimensional spacetime on which the gauge fields live. In the non-Abelian case each loop lives in an N index Chan-Paton state of an open and closed string theory within the world volume of N coincident Dp-branes. Note that the short-distance potential between semiclassical sources lying in the world volume of a single D-brane, or coincident D-branes, is independent of the non-Abelian nature of the gauge theory if we neglect interactions: for free strings, the colorless amplitude will simply scale as N^2 for N coincident D-branes. In what follows, we derive an expression for the spatial correlation function of a pair of Wilson loops lying within the world volume of a single D-brane in a generic flat spacetime background of the bosonic open and closed string theory. We extract from this expression a prediction for the short-distance potential between slow moving sources in the world volume gauge theory with small relative velocity $v=\tanh u \simeq u$ and relative position r. This gives

$$V(r,u)=-(d-2)\frac{1}{r}[1+O(z^2)+O(uz/\pi)+O(u^2)]. \tag{3}$$

The subleading terms in the potential will be obtained in a systematic expansion for small velocities and short distances valid down to distances of order $r_{\rm min}^2=2\pi\alpha' u$. They are succinctly expressed as a convergent power series in powers of the dimensionless variables $z=r_{\rm min}^2/r^2$, uz/π, and u^2. Note that the potential is a quantum effect accounting for the fluctuations about the minimum action surface which determines the saddle point of the string path integral. It is time reversal invariant [17]: the power series only contains even powers of u. The numerical coefficient of the static term is a measure of the number of degrees of freedom in the theory describing the short-distance dynamics. As with the Lüscher term in the long-distance potential between sources in a gauge theory, the static coefficient is free of both string coupling constant and string cutoff dependence. In a theory with supersymmetry, the leading static term will be absent but there is a corresponding velocity-dependent potential [13].

In this paper, we will give a prescription for the pair correlation function of macroscopic loop observables $M(C_i)$, $M(C_f)$ in bosonic open and closed string theory using a covariant path integral technique for one-loop string amplitudes developed by Polchinski [7], which determines unambiguously their normalization. This technique was applied to the covariant path integral for off-shell closed strings—amplitudes with macroscopic holes in the world sheet mapped to fixed curves in spacetime, in Ref. [8]. Explicit results were obtained for pointlike boundary states but an implementation of boundary reparametrization for finite sized loops directly in the path integral has been lacking so far. We note that there exists a Becchi-Rouet-Stora-Tyotin (BRST) analysis of boundary reparametrization invariance (see, for example, Ref. [18]) but BRST methods are unsuitable for addressing issues related to the normalization of the vacuum amplitude. The path integral implementation of boundary reparametrization invariance we will give is adapted from the work of Cohen *et al.* [8], and also from the earlier works [31,6,7]. We define the pair correlation function of macroscopic loop observables as the covariant string path integral

$$\langle M(C_i)M(C_f)\rangle \equiv \int_{[C_i,C_f]}\frac{[dg][dX]}{\mathrm{Vol(gauge)}}e^{-S[X,g_{ab};\mu_0,\lambda_0^{(i,f)}]}, \tag{4}$$

a reparametrization invariant sum over world sheets of cylindrical topology terminating on fixed boundary curves C_i, C_f, which are taken to lie in the world volume of a Dp-brane in flat spacetime. We will gauge both world-sheet diffeomorphisms and Weyl transformations of the world-sheet metric. Our results are derived for string theory in $d=26$ critical spacetime dimensions although they could be adapted to generic conformally invariant backgrounds of string theory with c_m matter fields coupled to a Liouville field, with total central charge equal to the critical dimension [16], as outlined in Sec. III B. $S[X,g_{ab}]$ is the bosonic Brink–Di Vecchia–Howe–Deser–Zumino action [14] plus appropriate bulk and boundary terms as necessary to preserve diffeomorphism and Weyl invariance. We compute quantum fluctuations about a saddle point describing a surface of minimum action stretched between coplanar loops of fixed length L_i, L_f, with spatial separation R. The short-distance potential between sources is extracted from the long loop length limit of this amplitude: L_i, $L_f\rightarrow\infty$ with R held *fixed*.

It should be noted that the one-loop amplitude with macroscopic boundaries is free of any coupling constant dependence. Corrections to the leading term in string perturbation theory are $O(g_{\rm open})$ from splitting, and $O(g_s)$ and higher order from open and closed string loops. Here $g_s = g_{\rm open}^2$, where $g_{\rm open}$ is identified with the Yang-Mills coupling, and gravitational corrections enter at $O(g_s^2)$ as closed string loops—suppressed at weak coupling. Nevertheless, even in Dp-brane backgrounds where the gauge fields live in $p<d-1$ spatial dimensions, evidence for the higher-dimensional string theory in which the gauge fields live is present in the numerical coefficient of the leading term in the short-distance potential. The reason is that the worldsheet fluctuates in all of the spacetime dimensions—both parallel and transverse to the world volume of the D-brane. From the viewpoint of a non-Abelian gauge theory, the transverse fluctuations arise from scalar fields in the adjoint representation of the gauge group.

We begin in Sec. II with a discussion of classical boundary reparametrization invariance, explaining its implementation in the string path integral. A boundary state in the bosonic string is specified by an embedding and an einbein. For fixed embedding of the loops, we give a boundary diffeomorphism invariant prescription for the measure in the path integral, summing over reparametrizations of the boundary. The gauge fixed path integral is derived in detail in Sec. III A. We determine the normalization of the path integral as in Polchinski's analysis of the torus amplitude in the bosonic string theory [7], extended to string amplitudes with boundaries [6,8,21]. For completeness, we retain the Liouville dynamics although our main interest is in string theory in the critical spacetime dimension. Sec. III B is an aside explaining how this analysis can be applied to Polyakov strings with generic conformally invariant backgrounds for the Liouville field following Ref. [16]. Readers whose main interest is in the potential calculation for string theory in the critical spacetime dimension can skip this subsection. The modifications to the pair correlation function for generic boundary conditions pertaining to slow-moving sources in relative motion within the world volume of the D-brane is derived in Sec. III C, an analysis similar to the scattering of slow moving D-branes in the bulk transverse space [19,20,17].

The computation of the potential between slow moving sources at short distances is given in Sec. IV. We consider heavy sources in the gauge theory in relative collinear motion with $r^2 = R^2 + v^2\tau^2$, $v << 1$, thus giving a simple realization of coplanar loops while mimicking nonrelativistic straight line trajectories in the Euclideanized X^0, X^p plane. Here r is their relative position, and τ is the zero mode of the Euclideanized time coordinate X^0. We will compute the Minkowskian potential for two sources in relative collinear motion with nonrelativistic velocity $v << 1$ for small separations r. In Sec. IV A, we extract the short-distance potential between two point sources traversing closed curves in spacetime for small separations r from the large loop length limit of the pair correlation function of Wilson loops. The scattering plane X^0, X^p can be wrapped into a spacetime cylinder by periodically identifying the coordinate X^0. Then the closed world lines of sources are loops singly wound about this cylinder, where the relative position of the sources at proper time τ is $r(\tau)$. We identify these closed world lines with Wilson loops. Define the effective potential as follows:

$$\langle M(C_i)M(C_f)\rangle = -i \lim_{T\to\infty}\int_{-T}^{+T} d\tau V_{\rm eff}[r(\tau),u], \qquad (5)$$

where we have taken the large loop length limit $L_i \simeq L_f \simeq T \to \infty$, with R held fixed. The dominant contribution to the potential between sources at short distances is from the massless modes in the open string spectrum. Restricting to these modes, we can express the potential as a double expansion in small velocities and short distances [20] with the result

$$V(r,u) = -\frac{\tanh(u)/u}{r(1+uz/\pi)^{1/2}}\left\{ (d-2)\frac{\gamma[\frac{1}{2},(\pi+uz)/z]}{\Gamma(\frac{1}{2})} + O[z^2/(1+uz/\pi)^2]\right\}, \qquad (6)$$

where z is a dimensionless scale factor, $z = r_{\rm min}^2/r^2$, and $r_{\rm min}$ is the minimum distance that can be probed in the small velocity expansion at short distances [19]: $r_{\rm min}^2 = 2\pi\alpha' u$. $\gamma[\frac{1}{2},(\pi+uz)/z]$ is the incomplete gamma function. The resummation and systematics of the small velocity expansion are discussed in Sec. IV A. The scale factor z determines the magnitude of the velocity-dependent corrections and, therefore, the accuracy of the expansion. For a given accuracy, with fixed z value, we can probe arbitrarily short distances r by simultaneously adjusting the velocity u. Self-consistency of the double expansion implies, however, an upper bound on the relative velocity $u \leqslant u_+$, thereby determining the regime of validity for the small velocity approximation. It should be noted that the leading terms in the potential can be obtained without use of the small velocity expansion. The potential is universal: independent of the dimensionality of the higher-dimensional D-brane, the geometrical parameters of the loop configuration, and the string scale cutoff. We note that there is no evidence for a departure from analyticity in the form of the potential between point sources in the bosonic string at short distances. The phase transition found in the large d analysis of a class of phenomenological string models including the Nambu-Goto string [22] appears to be a large d artifact.

D0-branes are pointlike spacetime topological defects present in the generic background of the open and closed bosonic string theory. In Sec. IV B, we note that the short-distance potential between two static D0-branes in bosonic string theory gives a linear interaction $V_{\rm D0\text{-}brane} = -(d-2)r/2\pi\alpha'$. The static potential is the shift in the vacuum energy due to a constant background electromagnetic potential, but with vanishing electric field strength [5,19,17]. The D0-branes are assumed to have fixed spatial separation in the direction X^{d-1}, and to be in relative motion with nonrelativistic velocity v in an orthogonal direction X^d [19,20,17]. The systematics of the small velocity short-distance double expansion, and the value for the minimum distance probed in the scattering of D0-branes, is identical to the results obtained in Sec. IV A in general agreement with previous work.

The bosonic string has a tachyon, formally suppressed in obtaining this result, which must be stabilized in order to obtain a theory with a sensible ground state (see, for example, the recent discussion in Ref. [23]). Alternatively, it can be eliminated from the spectrum of physical states, as is possible in the fermionic type-I and type-II string theories [17]. Evidence for distance scales in string-M theory shorter than the string scale down to the eleven-dimensional Planck length was originally observed in the form of the velocity-dependent potential between D0-branes in relative motion in tachyon-free backgrounds of type-II string theory [19,20,17]. D-branes correspond to BPS states in the type-II supergravity-Yang-Mills theory, solitons with masses of $O(1/g)$. The observation that solitons with masses of order $1/g$ can probe shorter distance scales than ordinary field theory solitons is originally due to Shenker [24]. Our result illustrates this principle directly in the gauge theory on the world volume of a D-brane in the bosonic string. Stated in complete generality for open and closed string theories at weak coupling: a Dirichlet boundary, or Wilson loop, can probe distance scales arbitrarily shorter than the string scale, whether in the world volume of the D-brane or in the bulk space orthogonal to the brane. We conclude with a brief discussion of the implications of our result in the broader context of gauge theory in generic backgrounds of string-M theory.

II. BOUNDARY REPARAMETRIZATION INVARIANCE

Following Cohen *et al.* [8], the tree correlation function for a pair of macroscopic string loops can be represented as a path integral over embeddings and metrics on world sheets of cylindrical topology terminating on fixed curves C_i, C_f, which lie within the world volume of a D-brane:

$$\langle M(C_i)M(C_f)\rangle=\int_{[C_i,C_f]}\frac{[dg][dX]}{\mathrm{Vol}[\mathrm{gauge}]}\times e^{-S_P[X,g_{ab}]-\mu_0\int d^2\sigma\sqrt{g}}, \tag{7}$$

where S_P is the bosonic Brink–Di Vecchia–Howe–Deser–Zumino action [14] on a surface with boundaries terminating on fixed curves. Note that the amplitude is free of the string coupling, since the Euler characteristic χ equals zero, and the boundary cosmological constant terms have been eliminated in favor of the bulk term since these are not independent Lagrange multipliers on a surface of cylindrical topology. In this section, we discuss the boundary conditions on the world-sheet fields which determine the saddle point of the path integral about which we are to compute quantum fluctuations.

Begin by considering the boundary conditions on the embedding coordinates. Setting the variation of the classical action with respect to the X^M to zero on the boundary yields

$$(\delta X^M)\hat{n}^a\partial_a X_M|_{\partial M}=0,\quad M=0,\ldots,d-1. \tag{8}$$

We will require boundary reparametrization invariance of the amplitude: each point on the physical boundary $\mathcal{C}$ is identified with a point on the piecewise continuous world-sheet boundary ∂M, but only up to a boundary reparametrization. Classically, this is most succinctly expressed as the modified Dirichlet boundary condition on the embedding functions [6] $\delta X^\mu|_{\partial M}\propto\hat{t}^a\partial_a X^\mu|_{\partial M}$, $\mu=0,\ldots,p$, and zero Dirichlet boundary conditions $X^m|_{\partial M}=0$, $m=p+1,\ldots,d-1$ in directions orthogonal to the brane volume. We can replace Eq. (8) with the equivalent condition

$$\hat{n}^a\hat{t}^b\partial_a X^\mu\partial_b X_\mu|_{\partial M}=0,\quad \mu=0,\ldots,p. \tag{9}$$

Note that upon imposition of the modified Dirichlet boundary condition on all d coordinates of a *space-filling* D-brane, the intrinsic world-sheet metric g_{ab} satisfies the same classical equation of motion as the embedding metric $\gamma_{ab}=\partial_a X^M\partial_b X^M$, summing on $M=0,\ldots,d-1$. As a consequence, under the mapping of the world-sheet boundary to fixed curves in the world volume of the D-brane, classically, the physical length of any closed curve is identified with the parameter length of a corresponding hole on the string world sheet.

Let σ^1 be the circle variable parametrizing any hole on the world sheet, and $\hat{e}=\sqrt{\hat{g}}|_{\partial M}$ be the einbein on the boundary, with fiducial metric $\hat{g}$. The metric on an arbitrary surface with cylindrical topology can be brought to the fiducial form, $ds^2=l^2(d\sigma^1)^2+(d\sigma^2)^2$, where $0\leqslant\sigma^1\leqslant1$, $0\leqslant\sigma^2\leqslant1$, and the area of the surface equals l. A reparametrization of the boundary $\Sigma\in\mathrm{Diff}_{\partial M}$, is a one-to-one invertible mapping of holes on the world sheet into corresponding fixed curves in spacetime

$$\Sigma[X_\mu(\sigma^1)|_{(i,f)}]=\tilde{x}_\mu^{(i,f)}[f^{(i,f)}(\sigma^1)]\qquad 0\leqslant\sigma^1\leqslant1, \tag{10}$$

Thus, the $\tilde{x}^{(i,f)}(\sigma^1)$ are *fiducial* maps of the boundaries of the world sheet into the spacetime curves C_i, C_f, and the $f^{(i,f)}$ are arbitrary diffeomorphisms of σ^1 parametrizing the corresponding holes on the world sheet.

The path integral sums over quantum fluctuations about a classical background determined by an extremum of the action. We look for minimum action configurations in the classical phase space of the Polyakov string. We separate each X into a piece $\bar{x}$ which solves the classical equation of motion with fiducial metric $\hat{g}$ and assumes the functional form $\tilde{x}^{(i,f)}$ on the boundary, and a quantum fluctuation which satisfies the zero Dirichlet condition. In directions orthogonal to the worldvolume of the D-brane, the $\tilde{x}^{(i,f)}$ are identically zero. Expanding the classical action in a complete set of modes

$$x_n^{(i,f)}=\int_0^1 d\sigma^1 x^{(i,f)}e^{2\pi in\sigma^1},\qquad n\in\mathrm{Z}, \tag{11}$$

spanning the classical phase space of boundary configurations, gives [8]

$$S_P[\overline{x};\hat{g}]=\frac{1}{4\pi\alpha'}\sum_{n=-\infty}^{\infty}\frac{2n\pi}{\sinh(2n\pi/l)}\times[(|x_n^i|^2+|x_n^f|^2)\cosh(2n\pi/l)-2\mathrm{Re}(x_n^i\cdot x_n^{f*})]. \tag{12}$$

Since our interest is in the large loop length limit, where the dynamics is hopefully universal and independent of the detailed geometric parameters of the loops, we make a judicious guess for minimum action configurations, $\tilde{x}^{(i,f)}$, obtaining the saddle point action from Eq. (12). A simple case is a pair of circular Wilson loops of uniform radius $L/2\pi$ separated by a distance R. Align the circles parallel to the X^p, X^0 plane, with their axis in a perpendicular direction. Here X^0 is a Euclidean coordinate. The minimum area world sheet is a catenoid [25]:

$$\overline{x}=[a\cos(2\pi\sigma^1),a\sin(2\pi\sigma^1),h(a)],$$
$$a^2=(X^p)^2+(X^0)^2, \tag{13}$$

with $L'/2\pi\leqslant a\leqslant L/2\pi$. The radial parameter a is related to the height of the catenoid $h(a)$ by the equation $a=(L'/2\pi)\cosh[2\pi h(u)/L']$. $L'/2\pi$ is the minimum radius of the cross section for the catenoid. It is straightforward to evaluate the Polyakov action for this surface. Consider the maps that must be included in the sum over reparametrizations of the world-sheet boundary for this configuration of loops. In general, this is a sophisticated problem in the representation theory of the group Diff(S^1). However, in the large loop length limit, the analysis is rather simple since winding number one maps with no self-intersections are energetically favored.

This feature of the large loop length dynamics is straightforwardly captured by considering the simple problem of summing over the reparametrizations of loops with one or more marked points. For such maps, the sum over reparametrizations of the boundary is easily implemented in closed form prior to taking a large loop length limit. For notational ease, let λ denote the circle variable, σ^1. We consider nonintersecting curves with the following characteristics: each $C^{(i,f)}$ is piecewise smooth with K straight line intervals of equal length $s_\alpha^{(i,f)}$ and K turning points, or corners, $\lambda_\alpha^{(i,f)}$, $\alpha=1,\ldots,K$. Any curvature on the boundary of the world sheet, if present, is permitted only at the corners. As can be seen from the Gauss-Bonnet theorem, this would induce a non-vanishing Euler characteristic $\chi_{\mathrm{corner}}=-\sum_{I=1}^2\sum_{\alpha=1}^K\delta_\alpha^{(i,f)}/2\pi$, and consequently a dependence on the string coupling constant in the amplitude. The angle terms arise from the delta function in the geodesic curvature at the corners. The *bulk* curvature is, however, required to be smooth: this implies that if we consider loop configurations with corners, it is convenient to choose the loops to be *coplanar*. For rectangular, or right-angled, loops, the turning angle, $\delta_\alpha^{(I)}=\pm\pi/2$, for every I, α. The simplest closed loop with net turning angle 2π is a rectangular loop with four edges, $K=4$. The example shown in Fig. 1 is a pair of coplanar nested rectangular loops, with $I=2$, $K=4$. The net turning angle for *both* loops vanishes. Thus, $\chi_{\mathrm{corner}}=0$ for C_i, C_f, and there is no dependence on the string coupling constant in the pair correlation function. It should be noted that, in general, the presence of corners would be a violation of boundary Weyl invariance giving a correction to the Liouville action [26]. *Any* pair of coplanar nested loops with arbitrary numbers of edges having net turning angle zero gives a Weyl invariant loop configuration with a well-defined saddle configuration: the stretched world sheet in the plane containing the loops. Smooth loops in this same class are the boundaries of an annulus—a pair of coplanar nested circular loops

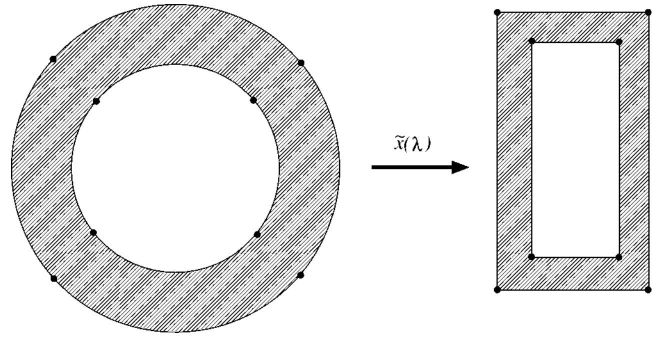

FIG. 1. $\tilde{x}(\lambda)$ is a map from the boundaries of the annulus to a pair of coplanar nested rectangular loops which lie within the worldvolume of a D-brane.

$$\overline{x}=[(R_0+R\sigma^2)\cos(2\pi\sigma^1),(R_0+R\sigma^2)\sin(2\pi\sigma^1)], \tag{14}$$

where $L_i=2\pi R_0$, $L_f=2\pi(R_0+R)$, and R is their separation in the radial direction. Periodically identifying Euclidean time, X^0 is the angular, and X^p the radial, direction. Now take the large loop length limit. Comparing with our discussion of the cylindrical spacetime in the introduction, the circular loops correspond to the closed worldlines of static sources with fixed spatial separation R. From Eq. (12), and in the limit $l\to\infty$, we obtain the saddle point action $S_P(\overline{x,\hat{g}})=-R^2l/4\pi\alpha'$. It is easy to verify an identical result for a pair of coplanar nested right-angled loops with arbitrary numbers of edges.

III. MACROSCOPIC LOOP CORRELATION FUNCTION

We now present the derivation of the closed string amplitude linking fixed curves C_i, C_f of length L_i, L_f, in an embedding spacetime with metric $\gamma_{ab}=\partial_a X^\mu\partial_b X_\mu$, and spatial separation R. We perform the sum over world sheet metrics using an idea taken from Cohen *et al.* [8]. We begin with the integration over all embeddings (X^m,X^μ), with fixed fiducial bulk metric, and fixed fiducial einbeins on the parameter boundary. We choose a fiducial metric on the world sheet, $ds^2=\hat{g}_{ab}d\sigma^a d\sigma^b$, with $\hat{e}=\sqrt{\hat{g}}$. Next we sum over world sheet metric deformations that leave fixed the parametrization of the boundary. Finally, we perform an integral over the ''boundary data,'' $\{e(\lambda;l_\alpha^{(i,f)})\}$, summing $\alpha=1,\ldots,K$, for all $2K$ intervals, restoring boundary reparametrization invariance. This last sum is defined as follows (see also the related ideas in Ref. [27,26]).

A boundary state is specified by an einbein and an embedding function $(e,\tilde{x}^{\mu})$. The embedding function is specified by our choice of saddle configuration, but we wish to leave its parametrization unfixed. The sum over einbeins implements reparametrization invariance on the boundary, and we must divide by the volume of the group of boundary diffeomorphisms $\mathrm{Diff}_{\partial M}$. Thus, we need a reparametrization invariant measure for the path integration over einbeins. The unique choice is Polyakov's quadratic form for metric deformations [31], restricted to any given boundary interval. On any interval, a boundary reparametrization $\Sigma \in \mathrm{Diff}_{\partial M}$, acts as

$$\Sigma[X_{\mu}|_{s_{\alpha}^{(i,f)}}]=\tilde{x}_{\mu}^{(i,f)}[f_{\alpha}^{(i,f)}(\lambda)] \quad \lambda_{\alpha-1}\leqslant\lambda\leqslant\lambda_{\alpha}, \quad \alpha=1,\dots,K, \tag{15}$$

where the λ_{α} are points in the range $0\leqslant\lambda\leqslant1$, and λ varies continuously with $\lambda_0=0$ identified with $\lambda_K=1$. Thus, $\tilde{x}_{\mu}^{(i,f)}(\lambda)$ is the fiducial map of the αth interval on the circle into the αth interval of the curve $C_{(i,f)}$, and $f_{\alpha}^{(i,f)}(\lambda)$ is the corresponding diffeomorphism of λ on the αth interval of the circle. Schematically, the path integration over quantum fluctuations due to an arbitrary diffeomorphism of the world sheet has been decomposed:

$$\frac{1}{\mathrm{order}(D)}\int\frac{[d\,\delta X][d\,\delta g]}{\mathrm{Vol}[\mathrm{Diff}]}\rightarrow\int\frac{[d\,\delta e_I(\lambda,\hat{e})]}{\mathrm{Vol}[\mathrm{Diff}_{\partial M}]} \times\int_{[\hat{e}]}\frac{[d\,\delta g]}{\mathrm{Vol}[\mathrm{Diff}_M]}\int_{[\hat{g};\hat{e}]}[d\,\delta X], \tag{16}$$

where $\mathrm{Vol}[\mathrm{Diff}_M]$ denotes the volume of the group of diffeomorphisms vanishing on the boundary, and $\mathrm{Vol}[\mathrm{Diff}_{\partial M}]$ that of the group of boundary diffeomorphisms. We divide by the order of the subgroup of the disconnected component of the diffeomorphism group, $\tilde{\mathrm{D}}$: discrete diffeomorphisms of the world sheet left invariant under the choice of conformal gauge [7]. Thus, a factor of 2 in the denominator corrects for the twofold invariance of the measure of the path integral under the diffeomorphism

$$\sigma^1\rightarrow-\sigma^1. \tag{17}$$

This symmetry will be left invariant under the gauge fixing of reparametrizations of the world sheet to be described below. The measure for embeddings and metrics do not individually respect Weyl invariance but, in critical string theory, their combination is Weyl invariant, and we therefore divide through by the volume of the Weyl group. In what follows, we will make this gauge fixing procedure explicit.

A. Gauge fixing reparametrizations

The gauge fixing of world-sheet metrics and the path integration of metrics and embeddings proceeds as in Ref. [7], except that all harmonic functions on the world sheet, scalars, and vectors, (X,η_a), are orthogonally decomposed $\Psi=\bar{\psi}+\psi'$. The ψ' vanish on the boundary, and the $\bar{\psi}$ are harmonic functions taking values $\bar{\psi}|_{\partial M}=\tilde{\psi}$ on the boundary. The $\bar{x}$ determine the saddle point configuration as described in the previous section. In general, we will allow for fluctuations of the world-sheet fields on ∂M, subject only to the constraint that they preserve the normal direction to the brane, the fixed embedding of the spacetime curves, and the smoothness condition at any corners of the boundary, if present. We will assume the reader is familiar with Ref. [7] and simply assemble the different contributions to the path integral.

Begin with the integration over embedding functions with fixed fiducial world sheet metric. We choose nested coplanar loops with physical lengths L_i, L_f and fixed spatial separation R. For each of the d scalar degrees of freedom, normalizing the path integration over harmonic functions vanishing on the boundary as in Refs. [7,21] gives the result

$$e^{-R^2l/4\pi\alpha'}(\det{}'\Delta_0)^{-d/2}=e^{-R^2l/4\pi\alpha'}\left[\eta\left(\frac{il}{2}\right)\right]^{-d}, \tag{18}$$

where the determinant of the Laplacian on scalars is computed with the zero Dirichlet boundary condition, and l is the cylinder modulus defined by the fiducial metric $ds^2=l^2(d\sigma^1)^2+(d\sigma^2)^2$, $0\leqslant\sigma^1\leqslant1$, $0\leqslant\sigma^2\leqslant1$. With this choice, the area of the world sheet is normalized to l.

Next consider the integration over metric deformations vanishing on the boundary. As in Refs. [31,6,7,21], we isolate the dependence on symmetric traceless variations of the metric and divide out by the volume of the gauge group $[\mathrm{Diff}_M]_0$, diffeomorphisms of the world sheet continuously connected to the identity and vanishing on the boundary. Normalize the path integrations on the cylinder as in Refs. [7,21]:

$$\begin{aligned}\int[d\,\delta\hat{g}]e^{-|\delta\hat{g}|^2/2}&\equiv\prod_{\sigma}\int[d\,\delta\hat{g}]_{\sigma}e^{-|\delta\hat{g}|_{\sigma}^2/2}=1\\&=J_M(\phi,\hat{g})\int[d\,\delta\phi]_{e^{\phi}\hat{g}}\\&\quad\times\int[d\,\delta\eta]'_{\hat{g}}\int_0^{\infty}dl\,e^{-|\delta\hat{g}|^2/2},\end{aligned} \tag{19}$$

where $|\delta\hat{g}|^2$ is the quadratic form for metric deformations and $J_M(\phi,\hat{g})$ is the Jacobian from the change of variables to deformations of, respectively, the Liouville mode $\delta\phi$, diffeomorphisms $\delta\eta_a$, vanishing on the boundary, and the cylinder modulus l, computed in Ref. [7]. The basic assumption underlying Eq. (19) is the locality of the measure: the integral over elements $[d\,\delta g]$ is a product of integrals over elements $[d\,\delta g]_{\sigma}$ at fixed values of the world-sheet coordinate σ. The only reparametrization invariant local counterterm (free of derivatives of the world-sheet metric) is of the form $M\int d^2\sigma\sqrt{g}$, which can be absorbed in a renormalization of the bulk cosmological constant μ_0 present in the bare action given in Eq. (7). Thus, the Gaussian integral on the left-hand side of Eq. (19) can be set to unity at the cost of renormalizing μ_0 [7]. The same argument applies to any of the world-sheet fields $(\delta\eta^a, \delta\phi, \delta X)$. The final value of the renormalized bulk cosmological constant μ_R is set to zero at the end of the calculation, giving a manifestly Weyl invariant result for the bosonic string theory in the critical spacetime dimension $d=26$.

Taking into account the contributions of the conformal Killing vector and zero modes of the Laplacian on vectors Δ_1 the infinity from the integration over diffeomorphisms $[d\eta]'_{\hat{g}}$ vanishing on the boundary is canceled against the volume of the gauge group $[\mathrm{Diff}_M]_0$. The result is an integral over the cylinder modulus times the quantum functional integral for Liouville field theory

$$J_M(\phi,\hat{g})\int[d\delta\phi]_{e^\phi\hat{g}}e^{-S_L[\phi,\hat{g}]-S_{\mathrm{boundary}}[\phi,\hat{e}]}, \qquad (20)$$

where S_{boundary} includes any boundary terms necessitated by the world-sheet gauge symmetries. $S_L[\phi,\hat{g}]$ is the unrenormalized Liouville action [31]

$$S_L[\phi,\hat{g}]=\frac{d-26}{48\pi}\int_M d^2\sigma\sqrt{\hat{g}}[\tfrac{1}{2}\hat{g}^{ab}\partial_a\phi\partial_b\phi+\hat{R}\phi] -\mu_0\int_M d^2\sigma\sqrt{\hat{g}}e^{\phi}, \qquad (21)$$

with an integration norm given by

$$|\delta\phi|^2=\int d^2\sigma\sqrt{\hat{g}}e^{\phi}(\delta\phi)^2. \qquad (22)$$

We will treat the measure for the einbeins following [7,8]. Note that the quantum functional integral in Eq. (20) denotes all possible bulk and boundary deformations of the Liouville field. In particular, it receives corrections from the measure for diffeomorphisms on the boundary as is shown below.

We continue with the sum over metric deformations nonvanishing on the boundary, orthogonal to the modes summed in Eq. (19). The metric on the world sheet is $\hat{g}_{ab}e^{\phi}$, and the fiducial einbein induced on the boundary is $\hat{e}=\sqrt{\hat{g}}$. The length of either boundary in the cylinder metric $\int_0^1 d\lambda\,\hat{e}$, equals l. A variation of the einbein on the αth interval is the result of a diffeomorphism $\lambda\to f_\alpha^{(i,f)}(\lambda)$ and a possible shift $\delta\phi$ in the Liouville field. Thus,

$$\begin{aligned}&\left\{\{(\hat{e}+\delta e)[\lambda+\delta f_\alpha^{(i,f)}(\lambda)]\}\left(1+\frac{d}{d\lambda}(\delta f_\alpha^{(i,f)})\right)+\hat{e}\,\delta\phi\right\}e^{\phi}\\ &=\left\{\hat{e}(\lambda)+\hat{e}\frac{d}{d\lambda}\delta f_\alpha^{(i,f)}+\delta e(\lambda)+O(\delta f_\alpha^{(i,f)})^2+\hat{e}\,\delta\phi\right\}e^{\phi}\\ &=\{l_\alpha^{(i,f)}+\delta\rho_\alpha^{(i,f)}[f_\alpha^{(i,f)}(\lambda)]\}e^{\phi},\end{aligned} \qquad (23)$$

where $l_\alpha^{(i,f)}$ is the length of the αth interval of the corresponding hole on the world sheet, and $\delta\rho$ is a rescaling of the einbein which can always be absorbed in a shift of the Liouville field on the boundary. The restriction of the quadratic form for metric deformations to the boundary ∂M gives a measure on the tangent space to the space of einbeins on any given interval

$$\begin{aligned}|\delta e|^2&=\int d\lambda[\hat{e}(\lambda;l)]^{-1}[\delta e(\lambda;l)]^2\\ &=\int d\lambda\left[-\hat{e}(\lambda)(\delta f)\frac{d^2}{d\lambda^2}(\delta f)+\frac{\{\delta\rho[f(\lambda)]\}^2}{\hat{e}}\right],\end{aligned} \qquad (24)$$

where the zero mode $\delta\rho_0[f(\lambda)]$ is the functional change in the length of the interval induced by a diffeomorphism.[1] Normalizing the path integrals as in Eq. (19):

$$1\equiv\int[d\delta e]e^{-|\delta e|^2/2}=J_{\partial M}(\hat{e})\int[d\delta f]_{\hat{g}}[d\delta\rho]_{\hat{g}e^{\phi}}e^{-|\delta e|^2/2}, \qquad (25)$$

where the Jacobian $J_{\partial M}$ is obtained as before, from a change of variables to deformations of boundary diffeomorphisms δf and einbein rescalings $\delta\rho$. Since a rescaling of the einbein is absorbed by a shift of the Liouville field we can, with no loss of generality, set $\rho=0$. Consequently, the integration over $[d\delta\rho]$ can be consistently dropped from the path integral. The infinity from the integration over diffeomorphisms will be canceled by the volume of the gauge group of diffeomorphisms on the boundary $\mathrm{Diff}_{\partial M}$, which has no disconnected part. Combining this analysis of boundary deformations with the bulk deformations in Eq. (19), we can write

$$1\equiv\left[\frac{(A/2\pi)^{1/2}(\det\chi^{ab}\chi_{ab})^{1/2}(\det{}'\Delta_1)^{1/2}}{(\det Q_{ab}/2\pi)^{1/2}}\right]\times\prod_{I=1}^{2}\left(\det{}'\left[-\frac{1}{l_I^2}\frac{d^2}{d\lambda^2}\right]\right)^{1/2}\int[d\delta\phi]_{\hat{g}e^{\phi}}e^{-|\delta g|^2/2}. \qquad (26)$$

The factor in square brackets is the Jacobian J_M derived in Ref. [7]. A is the world-sheet area $A=\int d^2\sigma\sqrt{g}$, and the term in the denominator arises from conformal Killing vectors, if present. On the cylinder, the area, $A=l$, in the metric defined above, and we have a single conformal Killing vector. The functional determinant of the Laplacian on vectors is computed as in Refs. [7,8]. With the fiducial cylinder metric given above,

$$J_M=\frac{(l/2\pi)^{1/2}\left(\frac{2}{l^2}\right)^{1/2}\left[\left(\frac{1}{2}l^2\eta^4\left(\frac{il}{2}\right)\right)\right]^{1/2}}{(l^3/2\pi)^{1/2}}, \qquad (27)$$

[1]Note that the conformal class of the metric determines the length of any boundary circle, a modulus of the surface. Thus, rescalings of the fiducial einbein must be absorbed in a shift of the Liouville field on the boundary in order that the conformal class of the metric is left unchanged.

up to ϕ-dependent terms absorbed in the Liouville action.

The second term in Eq. (26) is the boundary Jacobian $J_{\partial M}$ the product of independent determinants for each of K intervals, s^I_α, $\alpha=1,\ldots,K$. Note that the diffeomorphism acts on each circle as a whole, but independently on each of the two boundaries. Since the boundaries have a common parameter length l we obtain

$$J_{\partial M}=\det{}'\left[-\frac{1}{l^2}\frac{d^2}{d\lambda^2}\right]=2l. \tag{28}$$

A similar path integration appears in the problem of obtaining the off-shell propagator for a relativistic point particle, a discussion of which appears in Ref. [8]. See, also, the ansatz for a scalar quark loop given in Ref. [27].

Assembling Eqs. (16), (18), (20), (21), and (26), our result for the pair correlation function of piecewise smooth macroscopic loops C_i, C_f at fixed separation R is

$$\langle M(C_i)M(C_f)\rangle=\int_0^\infty dl e^{-R^2 l/4\pi\alpha'}\left[\eta\left(\frac{il}{2}\right)\right]^{2-d}\times\int\frac{[d\delta\phi]_{\hat{g}e^\phi}}{\mathrm{Vol(Weyl)}}\; e^{S[\phi;\hat{g}]}, \tag{29}$$

where $S[\phi;\hat{g}]=S_L[\phi;\hat{g}]+S_{\mathrm{boundary}}$ is the action for the Liouville field including boundary terms. In the critical spacetime dimension $d=26$, the Liouville dynamics entirely decouples, and we can consistently set ϕ to zero in Eq. (29) while dividing out by the volume of the Weyl group.

B. Generic Liouville backgrounds

It is possible to consider the cases $c_{\mathrm{matter}}<25$ following the method in Ref. [16]. We require that the path integral expression for the loop correlation function preserve quantum conformal invariance. We begin by suppressing quantum fluctuations and restrict to the zero mode ϕ_0 noting that the classical equation of motion is that of a free scalar field in the regime $\phi_0\to-\infty$: the exponential potential is suppressed. We will preserve this asymptotic property in defining the quantum theory: the wave functions (operators) of Liouville conformal field theory are required to match smoothly to free field states in the $\phi_0\to-\infty$ regime, characterized by momentum and occupation number alone.

Quantum Liouville conformal field theory can be defined by a functional integral over a renormalized Liouville field ϕ_R, with conformally invariant free field norm

$$|\delta\phi_R|^2=\int d^2\sigma\sqrt{\hat{g}}(\delta\phi_R)^2. \tag{30}$$

The ansatz of Ref. [16] is that the effects of renormalization can be lumped in the potential, leaving a kinetic term for ϕ_R with the canonical normalization of a free scalar field theory. Thus, we write

$$\int [d\delta\phi]_{e^\phi\hat{g}}e^{-S_L[\phi,\hat{g}]+S_{\mathrm{boundary}}[\phi,\hat{e}]}=\int [d\delta\phi]'_{\hat{g}}\int_{-\infty}^{\infty}d\phi_0 e^{-S[\phi_R,\hat{g}]}, \tag{31}$$

where $S[\phi_R]$ includes all possible renormalizable terms in the bulk, and on the boundary, that preserve both diffeomorphism and Weyl invariance. Note that a corner anomaly is a spontaneous breaking of Weyl invariance on the boundary, contributing an additional term not included in $S[\phi_R,\hat{g}]$.

In a Weyl invariant theory, the renormalized action $S[\phi_R,\hat{g}]$ takes the general form

$$S[\phi_R,\hat{g}]=\frac{1}{8\pi}\int_M d^2\sigma\sqrt{\hat{g}}[\tfrac{1}{2}\hat{g}^{ab}\partial_a\phi_R\partial_b\phi_R+Q\hat{R}\phi_R]-\mu_R\int_M d^2\sigma\sqrt{\hat{g}}e^{\alpha\phi_R}-\sum_{I=1}^{2}\lambda_R^{(I)}\int_{C_I}d\lambda\hat{e}e^{\beta^{(I)}\phi_R}, \tag{32}$$

where Q, α, and $\beta^{(I)}$, are constants determined by the requirement [16] that every term in Eq. (32) be a dimension one primary field, in a conformal field theory of vanishing total central charge $c_m+c_\phi+c_{\mathrm{ghosts}}=0$. The renormalized bulk and boundary cosmological constants μ_R, $\lambda_R^{(I)}$ are arbitrary marginal couplings in the conformal field theory. With no loss of generality, we could set the boundary cosmological constant term on the cylinder to zero, retaining μ_R.

The only mode of ϕ_R that survives on the boundary is ϕ_0, and the modes ϕ_R' satisfy Dirichlet boundary conditions as in Refs. [6–8,21]. Then, conformal invariance requires

$$Q=\sqrt{\frac{c_m-25}{3}},\quad \alpha=(\pm\sqrt{c_m-1}-\sqrt{c_m-25})/2\sqrt{3}, \tag{33}$$

the upper sign matching the dimension of the cosmological constant operator as computed in a semiclassical $c_m\to-\infty$ saddle point evaluation of the path integral for the Liouville field ϕ [27,16]. We will not pursue these cases further since our main interest in this paper is string theory in the critical spacetime dimension which corresponds to the theory with $c_m=25$, $c_{\phi_R}=1$.

C. Generic boundary conditions

Following Refs. [19,20], it is an easy extension to compute the pair correlation function with boundary conditions pertaining to closed world lines for a pair of slow moving sources in relative motion. Consider a pair of coplanar nested rectangular loops with the plane of the loops aligned parallel to the (X^0,X^p) plane. X^0, X^p are both Euclidean coordinates. Let us rotate one of the loops relative to the other through an angle ϕ in the (X^0,X^p) plane, and take the large loop length limit $L_i\simeq L_f\simeq T\to\infty$, with R held fixed. Upon analytic continuation $X^0\to iX^0_M$, the loops may be interpreted as the closed world lines of slow moving scalar quarks in collinear motion at short distances r and with relative velocity $v=\tanh(-i\phi)<<1$.

Consider the boundary conditions on X^0, X^p. We leave the boundary conditions at one end-point fixed, and change the condition at the other end point to $X^p|_{C_f}=vX^0$. Parametrize the world sheet with open string end points $\sigma^2=0,1$ at boundaries C_i, C_f, respectively, and an open string loop parametrized $0\leqslant\sigma^1\leqslant 1$. Recall that σ^1 is identified with the fiducial circle variable λ defined in the previous section. A complete set of eigenfunctions of the scalar Laplacian is composed from the basis

$$\psi^{(\alpha)}_{(n_1,n_2)}=e^{2n_1\pi i\sigma^1}\sin([n_2+\alpha]\pi\sigma^2), \tag{34}$$

where α takes values $-iu/\pi$ or $1+iu/\pi$. The velocity has been parametrized as $v=\tanh u$ and we work in the small velocity approximation with $v\simeq u$. The remaining $d-2$ embedding coordinates satisfy the zero Dirichlet boundary condition as in Sec. III A. The functional determinant of Δ_0 takes the form

$$\det\Delta_0^{(\alpha)}=\prod_{n_1,n_2}\left[\left(\frac{4\pi^2}{l^2}\right)\left(n_1^2+\frac{(n_2+\alpha)^2l^2}{4}\right)\right], \tag{35}$$

where l is the cylinder modulus defined above, and $-\infty\leqslant n_1\leqslant\infty$, $n_2\geqslant 0$. This is computed using zeta function regularization as in Ref. [7]:

$$\ln\det\Delta=-\lim_{s\to 0}\frac{d}{ds}\sum_{n_1,n_2}\left[\left(\frac{4\pi^2}{l^2}\right)\left(n_1^2+\frac{(n_2+\alpha)^2l^2}{4}\right)\right]^{-s}. \tag{36}$$

The infinite sum over n_1 is expressed as a contour integral by a Sommerfeld-Watson transform. The contour $\mathcal{C}$ runs counterclockwise from $\infty+i\epsilon$ to $-\infty+i\epsilon$. The result is

$$-\lim_{s\to 0}\frac{d}{ds}\left(\frac{4\pi^2}{l^2}\right)^{-s}\oint_{\mathcal{C}}\frac{dz}{2\pi i}\times\sum_{n_2\geqslant 0}\left[z^2+\frac{(n_2+\alpha)^2l^2}{4}\right]^{-s}\cot(\pi z). \tag{37}$$

Writing the cotangent as $-i\cot(\pi z)=2/(1-e^{-2\pi iz})-1$, we can extract the contribution from the integral that is finite in the limit $s\to 0$:

$$2\sum_{n_2\geqslant 0}\ln\left|1-e^{-\pi l(n_2+\alpha)}\right|. \tag{38}$$

The term singular in the limit $s\to 0$ has a finite l-dependent remnant whose coefficient can be identified as the regulated vacuum energy of a complex scalar

$$\begin{aligned}-\pi lE_0&\equiv\lim_{s\to 0}\frac{d}{ds}\left(\frac{4\pi^2}{l^2}\right)^{-s}\oint_{\mathcal{C}}dz\sum_{n_2\geqslant 0}\left[z^2+\frac{l^2(n_2+\alpha)^2}{4}\right]^{-s}\\&=\lim_{s\to 0}\frac{d}{ds}\left\{\left(\frac{4\pi^2}{l^2}\right)^{-s}l^{1-2s}\zeta(2s-1,\alpha)\times\frac{[\Gamma(1-s)]^2}{\Gamma(2-2s)}2^{1-2s}\tan(\pi s)\right\}\\&=-\pi l[(\alpha-\tfrac{1}{2})^2-\tfrac{1}{12}].\end{aligned} \tag{39}$$

Combining Eqs. (38) and (39), and with $q=e^{-\pi l}$, gives the result

$$(\det\Delta^{(v)})^{-1/2}=q^{-1/12+[u^2/\pi^2-i(u/\pi)]/2}\prod_{n_2=0}^{\infty}[(1-q^{n_2-iu/\pi})\times(1-q^{n_2+1+iu/\pi})]^{-1}, \tag{40}$$

which can be written as the ratio of Jacobi theta functions. Setting $u=0$ and suppressing the $n_2=0$ term in the result recovers the cylinder determinant for a pair of real scalars with Dirichlet boundary condition $[\eta(il/2)]^{-2}$.

Substituting in Eq. (29), we obtain an analogous result for the pair correlation function of macroscopic loops in critical string theory with boundary conditions pertaining to sources in slow relative motion:

$$\langle M(C_i)M(C_f)\rangle=2\int_0^\infty dl e^{-R^2l/2\pi\alpha'}\eta(il)^{-22}\frac{\eta(il)e^{-u^2l/\pi}}{i\Theta_{11}(ul/\pi,il)}. \tag{41}$$

For convenience, we have rescaled $l\to 2l$ in the integral.

IV. SHORT DISTANCE POTENTIAL BETWEEN SOURCES

We will now compute the potential between heavy nonrelativistic sources in the gauge theory in relative collinear motion with velocity $v=\tanh u\simeq u$. Parametrize the closed world lines of the sources by the proper time variable τ the zero mode of the Euclidean embedding coordinate X^0. Let $r(\tau)^2=R^2+v^2\tau^2$ denote the relative coordinate of the two sources in the X^0, X^p plane, where R is their static separation. We express the amplitude as an integral over Minkowskian time $-i\tau$. The loops are identified with the closed world lines of a heavy quark-antiquark pair in the gauge theory. This computation is described in Sec. IV A. We emphasize that the open string theory results derived in Sec. IV A are *only* to be applied to the short-distance limit of the potential between heavy sources in the gauge theory.

D0-branes are pointlike spacetime topological defects in the bosonic string theory [17]. Following Refs. [19,20,17], in Sec. IV B we compute the short distance interaction between two D0-branes in the bosonic string theory obtaining a linear *repulsive* static interaction. The systematics of the small velocity and short-distance double expansion yields similar conclusions for the minimum distance as in Sec. IV A.

A. Wilson loops and a short distance 1/r potential

We define the potential between sources traversing fixed world lines, $V_{\rm eff}[r(\tau),u]$ as follows:

$$\langle \cdots \rangle = -i\int_{-T/2}^{T/2} d\tau V_{\rm eff}[r(\tau),u], \tag{42}$$

and take the limit $T\to\infty$, with r held fixed. Then,

$$V_{\rm eff}(r,u) = 4(8\pi^2\alpha')^{-1/2} \times \int_0^\infty dl e^{-r^2 l/2\pi\alpha'} l^{1/2}\eta(il)^{-21}\frac{\tanh(u)e^{-u^2 l/\pi}}{\Theta_{11}(ul/\pi,il)}. \tag{43}$$

In the limit of short distances, the amplitude is dominated by the exchange of the lowest lying modes in the open string mass spectrum. We therefore expand in powers of $e^{-2\pi l}$, organizing the integrand as an infinite summation over open string modes, and restrict to the lowest lying states. We suppress the leading contribution from the open string tachyon—absent in any stable background, and focus on the subleading contribution from massless open string modes. We will show that the short distance potential between sources in the bosonic string—analogous in some respects to a nonsupersymmetric background of the superstring, has a static remnant originating in the massless modes, a measure of the degrees of freedom determining the short-distance dynamics of Wilson loops. Consider

$$V_{\rm eff}(r,v) = -2(8\pi^2\alpha')^{-1/2} \times \int_0^\infty dl e^{-r^2 l/2\pi\alpha'} l^{1/2}\frac{\tanh(u)e^{-u^2 l/\pi}}{\sin(ul)} \times\{e^{2\pi l}+[22+2\cos(2ul)]+O(e^{-2\pi l})\}, \tag{44}$$

where the restriction to massless modes gives

$$V(r,u) = -2(8\pi^2\alpha')^{-1/2} \times \int_0^\infty dl e^{-r^2 l/2\pi\alpha'} l^{1/2}\frac{\tanh(u)e^{-u^2 l/\pi}}{\sin(ul)} \times[22+2\cos(2ul)]. \tag{45}$$

We will now assume small velocities and short distances, performing a double expansion in the variables r, u. The regime of validity for the small u expansion is determined by the behavior of the cosecant function. We can perform a Taylor expansion in the first half-period of its argument, $0 \leqslant ul < \pi$. Consider the corrections to this result from the integration domain $ul \geqslant \pi$. The sine function changes sign at every $n\pi$, $n\in Z^+$, so that the regions, $n\pi\pm\epsilon$, where the integrand is singular can be excised from the domain of integration. This leaves the intervals $n\pi+\epsilon \leqslant ul \leqslant (n+1)\pi - \epsilon$. For sufficiently small u values the oscillations in the integrand will be increasingly rapid, smearing out the integral [17]. The result can always be bounded, or evaluated by numerical integration, as a self-consistency check on the validity of the small velocity short-distance approximation. This check provides an upper limit, u_+, on the permissible velocities. With this restriction, the contribution from the domain $l>\pi/u_+$ can be dropped and we will suppress it in what follows. Upon Taylor expansion of the periodic functions in the integrand, the potential can therefore be written as

$$V(r,u) = -2(8\pi^2\alpha')^{-1/2} \times \int_0^{\pi/u_+} dl e^{-r^2 l/2\pi\alpha'} l^{-1/2} e^{-u^2 l/\pi}\tanh(u)/u \times\left[24+\sum_{k=1}^\infty C_k(ul)^{2k}+\sum_{k=1}^\infty\sum_{m=1}^\infty C_{k,m}(ul)^{2(k+m)}\right], \tag{46}$$

where the coefficients of the expansion in powers of ul take the form

$$C_k = \frac{(-1)^k 2^{2k+1}}{2k!}+\frac{48|B_{2k}|}{2k!}(2^{2k-1}-1),$$

$$C_{k,m} = (-1)^m 2^{2(m+1)}\frac{(2^{2k-1}-1)|B_{2k}|}{2k!2m!}, \tag{47}$$

and the B_{2k} are the Bernoulli numbers. Integrating over l gives a systematic expansion for the potential in powers of u^2/r^4. Let us define a dimensionless scaling variable $z = r_{\rm min}^2/r^2$, where $r_{\rm min}^2 = 2\pi\alpha' u$. The velocity-dependent corrections to the potential are succinctly expressed as convergent power series in the dimensionless variables, z, uz/π, and u^2:

$$V(r,u) = -\frac{1}{\Gamma(\frac{1}{2})}\frac{\tanh(u)/u}{r(1+uz/\pi)^{1/2}} \times\Bigg\{24\gamma[\tfrac{1}{2},(\pi+uz)/z] + \sum_{k=1}^\infty C_k\gamma[2k+\tfrac{1}{2},(\pi+uz)/z] \times[z/(1+uz/\pi)]^{2k} + \sum_{k=1}^\infty\sum_{m=1}^\infty C_{k,m} \times\gamma[2(k+m)+\tfrac{1}{2},(\pi+uz)/z] \times[z/(1+uz/\pi)]^{2(k+m)}\Bigg\}. \tag{48}$$

The $\gamma(\nu,(\pi+uz)/z)$'s are incomplete gamma functions. In writing Eq. (48) we have assumed $u/u_+ << 1$. Note that if the variable z is taken to zero, for distances $r^2 >> r_{\rm min}^2$, we recover the ordinary gamma functions $\Gamma(\nu)$. The potential has a static remnant in the bosonic string. Setting u to zero in Eq. (48) gives the simple result

$$V(r) = -(d-2)\frac{1}{r}, \tag{49}$$

where $d=26$ is the critical spacetime dimension of the bosonic string. The velocity-dependent corrections have an analog in the type I′ superstring [13]. An analogous static term is present in the contribution from the Neveu-Schwarz sector [13], prior to cancellation by other contributions to the vacuum amplitude [17]. It is evident from Eq. (48) that our result for the potential between slow moving sources holds for arbitrarily short distance scales lower than the string scale $r_{\rm min}^2 \sim 2\pi\alpha' u$, limited only by the domain of validity for the double expansion in small velocities and short distances.

Let us compare the short distance static potential with the known form of the heavy quark-antiquark potential in QCD at long distances, a regime described by the effective dynamics of a thin flux tube linking the sources. The usual route from the Wilson loop expectation value to the static heavy quark-antiquark potential in gauge theory is as follows. Consider a rectangular Wilson loop RT in the limit $T/R \to \infty$ with R held fixed. The long legs of the rectangle are interpreted as the proper time world lines of a heavy quark and antiquark, and the loop expectation value takes the form $a(T)e^{-V(R)T}$, with $V(R)$ interpreted as the static quark-antiquark potential at fixed spatial separation R. $a(T)$ is some function with slower fall off than an exponential. The reader may wonder why we considered a pair correlation function rather than extract the potential from the expectation value of a single rectangular loop, as is usual in gauge theory. The reason is Weyl invariance: the worldsheet spanning a single rectangular boundary loop has curvature singularities at the corners leading to Weyl anomalies which would render a covariant path integral quantization untenable. The large loop length limit hides this problem since the corners are pushed to $\tau \to \pm\infty$. The pair correlation function does not suffer from this problem. In particular, for any pair of coplanar nested right-angled loops we had a well-defined expression for the string path integral even *prior* to taking the large loop length limit.

The heavy quark-antiquark potential at long distances displays a confining linear plus attractive inverse power law behavior

$$V(r) = \alpha r + \beta + \frac{\gamma}{r} + O(1/r^2). \tag{50}$$

α and β are known to be nonuniversal constants. Of greater interest is the universal constant γ first obtained by Lüscher *et al.* using heat kernel methods in Ref. [9]: $\gamma = -(d-2)\pi/24$ in the effective theory of the Eguchi-Schild string [3]. Recall the model-independent argument for the coefficient of the $1/r$ term [10] (see, also, the discussion in Ref. [28]). Consider the quantum dynamics of a thin flux tube linking quark and antiquark as described by an effective field theory. Let d be the number of degrees of freedom. Now the fluctuations of a long thin flux tube in $d-1$ spatial dimensions are described by a two-dimensional nonrenormalizable effective field theory of $d-2$ collective modes, with universal behavior that of $d-2$ free scalar fields, each with vacuum energy equal to $\pi/24$. The $O(1/r^2)$ terms that are quartic and higher order in the collective fields are irrelevant. The vacuum energy arises from the infinite sum of free field harmonic oscillators in their ground state, with an independent sum for each of $d-2$ degrees of freedom. Irrelevant couplings to higher dimensional operators can induce interactions; they determine the nonuniversal constants α, β. Not surprisingly, this long-distance result for the potential is in agreement with our expression for E_0, the vacuum energy from each of $d-2$ free world-sheet scalars given in Eq. (39).

Our computation demonstrates that there is also a universal $1/r$ static potential at *short distances*: independent of the dimensionality of the D-brane world volume, the geometrical parameters of the loops, and the string scale cutoff. As we can see from Eq. (49), the numerical coefficient at short distances predicted by string theory differs from Lüscher's long-distance result. This may be interpreted in the effective field theory as a wave function renormalization for the Wilson loop observable at short distances, an effect which *cannot* be determined in a field theoretic analysis insensitive to boundary effects. Moreover, there is an *infinite* number of velocity-dependent corrections to the $1/r$ term which are *also* universal. We obtained these corrections by a systematic double expansion in small velocities and short distances, conveniently expressed as a convergent power series in dimensionless variables, $z = r_{\rm min}^2/r^2$, uz/π, and u^2.

Our results can also be considered within the more traditional context of phenomenological models for short distance nonperturbative dynamics in QCD (see, for example, Ref. [31], and references therein). The generic backgrounds for the Liouville field with $c_m < d$ described in Sec. III C could be of interest in this context. We note that recent work[2] on the short-distance potential between heavy sources in QCD has examined modifications of the potential at short distances originating in nonperturbative instanton effects [32].

B. Minimum distance and the short distance scattering of D0-branes

D0-branes are pointlike topological defects in spacetime present in the generic background of the bosonic string theory. Consider the potential between two D0-branes probed in their nonrelativistic scattering [19,29,20]. The D0-branes are assumed to have fixed spatial separation b in the direction X^{d-1}, and are in relative slow motion in an orthogonal direction X^d with velocity v [19]. At long distances their static interaction potential will take the Newtonian form. The effective potential at long distances is dominated by the exchange of the lowest lying states in the *closed* string spectrum. With no loss of generality, we can obtain the potential between two D0-branes as a special case of the general expression for the scattering of two Dp-branes $p<d$.

[2] We would like to thank M. Eides for bringing this work to our attention.

We will show that the systematics of the small velocity short-distance double expansion and the value for the minimum distance probed in the scattering, $r_{\min}$, is identical to the result obtained above, in general agreement with previous analyses [19,20,17].

Adapting the computation of the bosonic string annulus amplitude between static Dp-branes [30,17] to the boundary conditions pertinent to Dp-brane scattering, and restricting to massless modes gives

$$V_{\mathrm{D}p\text{-brane}}(r,u)=-V_p(8\pi^2\alpha')^{-(p+1)/2}\times\int_0^\infty dl e^{-r^2 l/2\pi\alpha'} l^{21-p/2}\left[\frac{\tanh(u)}{i\sin(-iu)}\times[22+2\cosh(2u)]\right]. \quad (51)$$

Notice that, unlike the expression for the short distance potential, a Taylor expansion of the periodic functions in the integrand for small velocities and long distances gives only $O(u^2)$ corrections to the static potential. The integration domain is unrestricted for small velocities. Performing the integration over l gives the simple result

$$V_{\mathrm{D}p\text{-brane}}(r,u)=-V_p(8\pi^2\alpha')^{-p/2}\Gamma\left(\frac{23-p}{2}\right)\times(2\pi\alpha')^{(23-p)/2}r^{p-23}[24+O(u^2)]. \quad (52)$$

Setting $p=0$ gives the Newtonian long distance interaction for D0-branes in a $d=26$ dimensional spacetime.

At short distances, we will find a crossover phenomenon analogous to what was found in the interaction potential of a Dp-brane with a Dp'-brane for dimensionalities, $p-p'\neq 0$ mod 4 [20]: the asymptotic long- and short-distance forms of the pair potential between Dp-branes in the bosonic string are *not* identical. Consider the expression for the effective potential due to the exchange of massless modes in the open string spectrum

$$V_{\mathrm{D}p\text{-brane}}(r,u)=-V_p(8\pi^2\alpha')^{-(p+1)/2}\times\int_0^\infty dl e^{-r^2 l/2\pi\alpha'} l^{-(p+1)/2}\times\left[\frac{\tanh(u)e^{-u^2 l/\pi}}{\sin(ul)}[22+2\cos(2ul)]\right]. \quad (53)$$

The small velocity short-distance double expansion can be performed as explained in the previous section. The result is the expression

$$V_{\mathrm{D}p\text{-brane}}(r,u)=-V_p(8\pi^2\alpha')^{-(p+1)/2}\tanh(u)/u\times\left\{24\gamma\left[-\frac{p+1}{2},(1+uz/\pi)/z\right]\times\left(\frac{r^2}{2\pi\alpha'}\right)^{(p+1)/2}(1+uz/\pi)^{(p+1)/2}+O(z^2,uz/\pi,u^2)\right\}, \quad (54)$$

where the velocity-dependent corrections are obtained in a systematic expansion in the same dimensionless variables z^2, uz/π, and u^2 defined above. For $r^2>>r^2_{\min}$, $z\rightarrow 0$, we recover the gamma functions $\Gamma(-\nu)$. Recall that gamma functions with negative argument can be defined by iterating the well-known identity $-\nu\Gamma(-\nu)=\Gamma(-\nu+1)$. We note that the short-distance static potential between D0-branes is linear, and *repulsive*

$$V_{\mathrm{D0\text{-}brane}}(r)=-(d-2)\frac{r}{2\pi\alpha'}+O(z^2,uz/\pi,u^2). \quad (55)$$

This result holds in a self-consistent small velocity short-distance approximation with corrections of $O(z^2,uz/\pi,u^2)$. It is valid for distances in the range $2\pi\alpha' u<<r^2\leqslant 2\pi\alpha'$ and velocities in the range $u<<u_+$, where the upper bound is estimated as described in Sec. IV A.

The static potential between D0-branes corresponds to the vacuum energy in a background of open string theory with constant electromagnetic potential A^{d-1}, but with vanishing electric field strength $E^{d-1}=\partial_0 A^{d-1}=0$ [5,19,17]. The potential is a measure of the shift in the vacuum energy relative to that in the background with no D-brane sources.

V. CONCLUSIONS

Our computation in open and closed string theory is performed at weak coupling in flat spacetime backgrounds and in the critical spacetime dimension. There is a supersymmetric analogue to this result which will be explored in forthcoming work [13]. We have demonstrated the validity of the double expansion in small velocities and short distances down to a minimum distance $r^2_{\min}=2\pi\alpha' u$, in general agreement with previous estimates [24,19,20,17]. Thus, string-M theory predicts an *infinite* number of velocity-dependent corrections to the potential between two heavy sources in relative slow motion in a gauge theory, the numerical coefficients of which are predicted by a systematic expansion. We are not aware of a comparable theoretical analysis which reliably probes this regime of QCD. We note that there is no evidence of nonanalytic behavior in the potential between sources at short distances, suggesting that the phase transition at short distances previously found in Refs. [22,27] is a large d artifact.

The numerical difference between the coefficient of the $1/r$ term we have found in the static potential at short distance and that given by Lüscher's effective field theory analysis for the QCD flux tube valid at long-distance scales deserves explanation. We should note that the flux tube picture is inherently a long-distance concept whose predictions cannot be naively extrapolated to short distances. Consider the $(d=26)$-dimensional bosonic string. At both long and short distances there is a proportionality factor in the $1/r$ potential which equals $d-2$, the number of transverse massless gluon modes. The short-distance static potential between heavy point sources in the gauge theory is a measure of fluctuations in the vacuum energy density. It would be gratifying if one could exploit the direct calculation of the short-distance potential from string theory given in this paper to explore nonperturbative physics associated with the QCD vacuum at short distances, a subject rich in conjecture and in open questions [31]. This deserves further study.

We conclude by noting that the characteristic D0-brane velocity in the supersymmetric theory is of order $u \simeq g_s^{2/3}$ [29], which implies a minimum distance $r_{\rm min}^2 \simeq \alpha' g_s^{2/3}$. It is interesting to note that at large N, the shortest distances that can be probed in the small velocity short-distance approximation are pushed down to $r_{\rm min}^2 = g_{\rm eff}^{2/3}/N$ [20], where $g_{\rm eff} = g_s N$. In the Introduction, we emphasized the importance of taking a large N limit in order to keep gravitational corrections to amplitudes in the open string sector suppressed at *long* distances; this was also an essential observation underlying Maldacena's conjecture [12]. We see now that taking the large N limit extends the regime of weakly coupled open and closed string theory *both* in the directions of longer- *and* of shorter-distance scales. We believe it would be of great interest to develop a systematic formulation of the large N limit of open and closed string theory.

ACKNOWLEDGMENTS

S.C. would like to thank M. Douglas, N. Drukker, M. Eides, S. Giddings, D. Gross, A. Hashimoto, D. Kabat, D. Kutasov, G. Moore, J. Polchinski, S.-J. Rey, S. Shenker, and H. Tye for their comments, and the Institute for Theoretical Physics and the Aspen Center for Physics for their hospitality. This work was supported in part by the National Science Foundation Grant No. NSF-PHY-97-22394.

[1] K.G. Wilson, Phys. Rev. D **10**, 2445 (1974).
[2] Y. Nambu, Phys. Lett. **80B**, 375 (1979); J. Gervais and A. Neveu, *ibid.* **80B**, 255 (1979); A. Polyakov, *ibid.* **82B**, 247 (1979).
[3] A. Schild, Phys. Rev. D **16**, 1722 (1977); T. Eguchi, Phys. Rev. Lett. **44**, 126 (1980).
[4] A.M. Polyakov, Phys. Lett. **103B**, 207 (1981).
[5] J. Dai, R. Leigh, and J. Polchinski, Mod. Phys. Lett. A **4**, 2073 (1989); R. Leigh, *ibid.* **4**, 2767 (1989); J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995).
[6] O. Alvarez, Nucl. Phys. **B216**, 125 (1983).
[7] J. Polchinski, Commun. Math. Phys. **104**, 37 (1986).
[8] A. Cohen, G. Moore, P. Nelson, and J. Polchinski, Nucl. Phys. **B267**, 143 (1986).
[9] M. Lüscher, K. Symanzik, and P. Weisz, Nucl. Phys. **B173**, 365 (1980).
[10] M. Lüscher, Nucl. Phys. **B180** [FS2], 317 (1981).
[11] S.J. Rey and Jungtay Yee, ''Macroscopic strings as heavy quarks in large N gauge theory and anti–de Sitter'' supergravity, hep-th/9803001; J. Maldacena, Phys. Rev. Lett. **80**, 4859 (1998).
[12] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[13] S. Chaudhuri and E. Novak, ''Supersymmetric Pair Correlation Function of Wilson Loops,'' hep-th/0002046, Phys. Rev. D (to be published).
[14] L. Brink, P. Di Vecchia, and P. Howe, Phys. Lett. **65B**, 471 (1976); S. Deser and B. Zumino, *ibid.* **65B**, 369 (1976).
[15] M. Peskin, Phys. Lett. **94B**, 161 (1980).
[16] F. David, Mod. Phys. Lett. A **3**, 1651 (1988); J. Distler and H. Kawai, Nucl. Phys. **B321**, 509 (1989).
[17] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998), Vols. I and II; Sec. 13.5.
[18] D. Birmingham and C.G. Torr, Phys. Lett. B **205**, 289 (1988).
[19] C. Bachas, Phys. Lett. B **374**, 37 (1996).
[20] M. Douglas, D. Kabat, P. Pouliot, and S. Shenker, Nucl. Phys. **B485**, 85 (1997).
[21] S. Chaudhuri, Phys. Rev. D **60**, 106007 (1999); hep-th/9907179.
[22] O. Alvarez, Phys. Rev. D **24**, 440 (1981).
[23] A. Sen and B. Zwiebach, J. High Energy Phys. **03**, 002 (2000) and references within.
[24] S. Shenker, ''Another length scale in string theory,'' hep-th/9509132.
[25] See, for example, E. Kreyszig, *Differential Geometry* (Dover, New York, 1991).
[26] C. Varughese and W.I. Weisberger, Phys. Rev. D **37**, 981 (1988).
[27] B. Durhuus, P. Olesen, and J. Petersen, Nucl. Phys. **B232**, 291 (1984).
[28] J. Polchinski and A. Strominger, Phys. Rev. Lett. **67**, 1681 (1991).
[29] D. Kabat and P. Pouliot, Phys. Rev. Lett. **77**, 1004 (1996); U. Danielsson, G. Ferretti, and B. Sundborg, Int. J. Mod. Phys. A **11**, 5463 (1996).
[30] J. Polchinski, S. Chaudhuri, and C. Johnson, ''Notes on Dbranes,'' hep-th/9602052; See, also, Sec. 8.7, Vol. I, Ref. [17].
[31] A. M. Polyakov, *Gauge Fields and Strings* (Harwood, Chur, 1987). For a recent overview, see talk given at ''Strings at the Millenium'' conference, Caltech, 2000.
[32] F. Gubarev, M.I. Polikarpov, and V.I. Zakharov, ''Short strings in the Abelian Higgs model,'' hep-th/9812030; ''Physics of the power corrections in QCD,'' review lectures by V.I. Zakharov at the ITEP winter school, hep-ph/9908292.